STABILITY OF THE FLOW OF A FILM OF LIQUID METAL,
PRODUCED BY AN INTENSE ELECTRON BEAM

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UDC 532.51.013.4

Hydrodynamic processes occurring in welding tanks will play an important role in electron-beam welding with deep melting, since the processes involved in the transfer of liquid metal in the working zone determine most of the defects appearing in welding seams (cavities in the body of a seam, fluctuations in the depth of melting along a seam), and, in many cases, the hydrodynamics determine the productivity of the operation. However, a complete investigation of hydrodynamic processes is greatly complicated by their interaction with heat transfer, vaporization, and interaction of metal vapor with the electron beam. For this reason, certain simplifications must be made at the first stage of the analysis. In this paper, as a first approximation we examine the flow of metal along the melting front in electron-beam welding with deep melting as a plane flow of a film of melted metal driven by gravity and the momentum of the electrons. We determine the stationary distribution function of the current and of the temperature corresponding to such a flow, and we investigate the hydrodynamic stability of this stationary flow relative to infinitesimal two-dimensional perturbations. The scheme of the interaction of the high-density electron beam with the metal, which leads to the formation of the liquid film, is shown in Fig. 1. The semibounded mass of metal is introduced with velocity V, oriented along the y axis; the boundary $y = \eta_0$ is the surface of the phase transition from the solid into the liquid state, and a phase transition from the liquid into the gaseous state under the action of the electron beam with energy density $q \sim 10^9$ W/m^2, oriented at an angle β to the surface of the metal and distributed uniformly along the entire x axis, occurs at $y = \eta$. The surface of the metal forms an angle δ with the horizontal. The process occurs in a vacuum. The interaction of the electron beam with the metal vapor is ignored.

1. Derivation of the System of Equations for the Amplitudes of the Perturbations. In dimensionless variables, for a two-dimensional flow we have the following Navier–Stokes equations, describing the motion of a viscous liquid:

$$\frac{\partial^2\psi}{\partial\tau\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=-\frac{\partial p_l}{\partial x}+\frac{gh\sin\delta}{U_m^2}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\Delta\psi, \tag{1.1}$$
$$\frac{\partial^2\psi}{\partial\tau\partial x}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}=\frac{\partial p_l}{\partial y}+\frac{gh\cos\delta}{U_m^2}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\Delta\psi,$$

where ψ is the stream function and p_l is the pressure in the liquid.

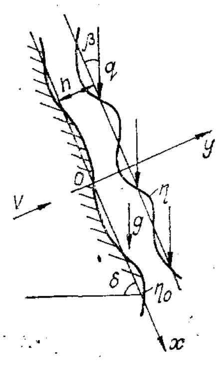

Fig. 1

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 99-105, March-April, 1984. Original article submitted March 14, 1983.

 0021-8944/84/2502-0256$08.50

In writing down (1.1), the average thickness of the liquid film h and the maximum magnitude of the x component of the flow velocity of the film U_m, attained at the upper boundary of the film with stationary flow, were used as the characteristic values of the length and velocity, respectively. Reynolds number $Re = U_m h/\nu$, where ν is the kinematic coefficient of viscosity of the melted metal.

Elimination of p_l from Eqs. (1.1) yields the equation for the stream function

$$\frac{\partial\Delta\psi}{\partial\tau}+\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}=\frac{1}{\mathrm{Re}}\Delta\Delta\psi. \tag{1.2}$$

The dimensionless heat-conduction equations for the liquid and solid phases are written as follows:

$$\frac{\partial t_l}{\partial\tau}\quad\frac{\partial\psi}{\partial y}\frac{\partial t_l}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial t_l}{\partial y}=\frac{1}{\mathrm{Re\,Pr}}\Delta t\ ; \tag{1.3}$$

$$\frac{\partial t_s}{\partial\tau}+v\frac{\partial t_s}{\partial y}=\frac{1}{\mathrm{Re\,Pr_s}}\Delta t_s. \tag{1.4}$$

Here $Pr = \nu/a_l$ is Prandtl's number for the melted metal; $Pr_s = \nu/a_s$, a_l and a_s are the coefficients of thermal diffusivity of the liquid and solid metal; $v = V/U_m$. The maximum temperature of the liquid metal T_m, attained at the upper boundary of the film in stationary flow, is used as the characteristic temperature.

The solutions of Eqs. (1.2)-(1.4) must satisfy a number of boundary conditions. These conditions have the following dimensionless form: "attachment" at $y = \eta_0$

$$\partial\psi/\partial y=0,\quad -\partial\psi/\partial x=v; \tag{1.5}$$

melting at $y = \eta_0$

$$t_l=T_{\mathrm{melt}}/T_m,\quad t_s=T_{\mathrm{melt}}/T_m, \tag{1.6}$$

where T_{melt} is the melting temperature of the metal; continuity of the energy flux at $y = \eta_0$

$$-\eta_{0x}\frac{\partial t_l}{\partial x}+\frac{\partial t_l}{\partial y}=\frac{\lambda_s}{\lambda_l}\left(-\eta_{0x}\frac{\partial t_s}{\partial x}+\frac{\partial t_s}{\partial y}\right)+\frac{h\rho V L_{\mathrm{melt}}}{\lambda_l T_m}, \tag{1.7}$$

where $\eta_{0x} = \partial\eta_0/\partial x$, λ_s and λ_l are the coefficients of thermal conductivity of the solid and liquid metal, ρ is the density of the metal (assumed to be the same for the solid and liquid phases), L_{melt} is the specific heat of fusion; the condition at a metal boundary at infinity

$$t_s=T_\infty/T_m\ \text{ at }\ y=-\infty, \tag{1.8}$$

where T_∞ = const; continuity of the flux of matter at $y = \eta$

$$\left(-\eta_x\frac{\partial\psi}{\partial y}-\frac{\partial\psi}{\partial x}-\eta_\tau\right)\frac{1}{\sqrt{1+\eta_x^2}}=\rho_v v_{vn}, \tag{1.9}$$

where $\eta_x = \partial\eta/\partial x, \eta_\tau = \partial\eta/\partial\tau$, ρ_v is the density of the metal vapor, scaled to the density of the liquid metal, v_{vn} is the normal component of the dimensionless velocity of the metal vapor; continuity of the tangential component of the momentum flux at $y = \eta$

$$-\frac{4}{\mathrm{Re}}\frac{\partial^2\psi}{\partial x\partial y}\frac{\eta_x}{1+\eta_x^2}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)\frac{1-\eta_x^2}{1+\eta_x^2}=q\sqrt{2m_e/(\bar{e}u_e)}\Big/(\rho U_m^2)\left(\frac{\sin 2\beta}{2}\frac{1-\eta_x^2}{1+\eta_x^2}+\cos 2\beta\frac{\eta_x}{1+\eta_x^2}\right), \tag{1.10}$$

where m_e and $\bar{e}$ are the mass and charge of an electron, u_e is the accelerating stress; continuity of the normal component of the momentum flux at $y = \eta$

$$p_l+\frac{2}{\mathrm{Re}}\frac{\partial^2\psi}{\partial x\partial y}\frac{1-\eta_x^2}{1+\eta_x^2}+\frac{2}{\mathrm{Re}}\left(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)\frac{\eta_x}{1+\eta_x^2}=p_v+\rho_v v_{vn}^2, \tag{1.11}$$

where p_l and p_v are the dimensionless pressures in the liquid and in the vapor of the metal; continuity of the energy flux at $y = \eta$

$$Q(\sin\beta+\eta_x\cos\beta)\frac{1}{\sqrt{1+\eta_x^2}}=\rho_v v_{vn}\left(\frac{v_{vn}^2}{2}+L\right)+\lambda\left(-\eta_x\frac{\partial t_l}{\partial x}+\frac{\partial t_l}{\partial y}\right)\frac{1}{\sqrt{1+\eta_x^2}}, \tag{1.12}$$

where $Q = q/(\rho U^3_m)$, $L = L_{vap}/U^2_m$, L_{vap} is the specific heat of vaporization of the metal, $\lambda = \lambda_l T_m/(h\rho U^3_m)$; equality of the temperatures of the liquid and metal vapor at $y = \eta$

$$t_l = t_v, \tag{1.13}$$

where t_v is the temperature of the metal vapor; the Mendeleev—Clapeyron equation

$$p_l = \rho_l R T_m t_v / U_m^2, \tag{1.14}$$

where R is the universal gas constant; the Clasius—Clapeyron equation

$$p_l = B/(\rho U_m^2) \exp[-L_{vap}/(RT_m t_l)] \quad \text{at} \quad y = \eta, \tag{1.15}$$

where $B = p_0 \exp[L_{vap}/(RT_b)]$, T_b is the boiling point of the metal at atmospheric pressure p_0; Jouguet's condition [1]

$$v_{vn} = a/U_m \text{ at } y = \eta, \tag{1.16}$$

where $a = \sqrt{\gamma R T_m t_v}$ is the velocity of sound in the metal vapor, and γ is the adiabatic index.

Equations (1.1), (1.3), and (1.4) have the following exact time-independent solution:

$$\psi_0 = A_1 y + B_1 y^2 - A_1/k_1 \exp(k_1 y) + D_1 x,$$
$$t_{l0} = A_2 + B_2 \exp(k_2 y), \quad t_{s0} = A_3 + B_3 \exp(k_3 y),$$

where $k_1 = Vh/\nu$, $k_2 = Vh/a_l$, $k_3 = Vh/a_s$, $B_1 = gh \sin\delta/(2VU_m)$. The values of the coefficients A_1, A_2, A_3, B_2, B_3, and D_1 are found from the boundary conditions, written in the stationary case at the boundaries $y = -\infty$, $y = 0$, and $y = 1$: $A_1 = \nu(g \sin\delta/V - \tau_e/\mu)/(VU_m \exp k_1)$, where $\tau_e = q\sqrt{2m_e/(\bar{e}u_e)} \cdot \sin\beta\cos\beta$, $\mu = \rho\nu$ is the dynamic coefficient of viscosity of the liquid metal; $A_2 = T_{melt}/T_m - (1 - T_{melt}/T_m)/(\exp k_2 - 1)$, $A_3 = T_\infty/T_m$, $B_2 = (1 - T_{melt}/T_m)/(\exp k_2 - 1)$, $B_3 = (T_{melt} - T_\infty)/T_m$, $D_1 = -V/U_m$.

The temperature T_m, the thickness of the liquid film h, and the velocity of inflow V are also found from the boundary conditions. These quantities depend on the energy density q, the angle β, and the thermophysical characteristics of the metal. Thus, the temperature T_m is determined from the transcendental equation

$$q \sin\beta = B \exp[-L_{vap}/(RT_m)][c_l(T_m - T_{melt}) + c_s(T_{melt} - T_\infty) + L_{melt} + L_{vap} + \gamma R T_m/2]/[(\gamma+1)\sqrt{RT_m/\gamma}],$$

where c_l and c_s are the specific heat capacities of the liquid and solid metal. The velocity of inflow of metal mass

$$V = q\sin\beta/\{[c_l(T_m - T_{melt}) + c_s(T_{melt} - T_\infty) + L_{melt} + L_{vap} + \gamma R T_m/2]\rho\}.$$

The thickness of the liquid film

$$h = \frac{a_l}{V} \ln\left(1 + \frac{c_l}{c_s}\frac{T_m - T_{melt}}{T_{melt} - T_\infty + L_{melt}/c_s}\right).$$

It can be shown, however, that stationary flow is unstable for some ratio of the parameters of the problem. We set

$$\psi = \psi_0 + \psi_1, \quad t_l = t_{l0} + t_{l1}, \quad t_s = t_{s_0} + t_{s_1}, \quad \eta = 1 + \eta_1,$$
$$\eta_0 = 0 + \eta_{01}, \quad p_v = p_{v0} + p_{v1}, \quad \rho_v = \rho_{v0} + \rho_{v1}, \quad t_v = t_{v0} + t_{v1},$$

where p_{v0}, ρ_{v0}, and t_{v0} are the characteristics of the metal vapor corresponding to the stationary process; quantities labeled with the index 1 are infinitesimal planar periodic perturbations of the following form:

$$\begin{aligned} \psi_1 &= f(y)\exp[i\alpha(x - c\tau)], & t_{l1} &= \varphi_l(y)\exp[i\alpha(x - c\tau)], \\ t_{s1} &= \varphi_T(y)\exp[i\alpha(x - c\tau)], & \eta_1 &= n\exp[i\alpha(x - c\tau)], \\ \eta_{01} &= n_0\exp[i\alpha(x - c\tau)], & p_{v1} &= p_1\exp[i\alpha(x - c\tau)], \\ \rho_{v1} &= \rho_1\exp[i\alpha(x - c\tau)], & t_{v1} &= t_1\exp[i\alpha(x - c\tau)], \end{aligned} \tag{1.17}$$

where α is the wave number and c is the phase velocity of the perturbation.

We substitute (1.17) into Eqs. (1.2)-(1.4). After linearizing, we obtain a system of ordinary differential equations for the amplitudes of the perturbations of the stream function and temperature of the liquid and solid phases

$$(u-c)(f''-\alpha^2 f)-u''f=-\frac{i}{\alpha \operatorname{Re}}(f^{IV}-2\alpha^2 f''+\alpha^4 f)+\frac{ik_1}{\alpha \operatorname{Re}}(f'''-\alpha^2 f'); \tag{1.18}$$

$$(u-c)\varphi_l-t'_{l0}f=-\frac{i}{\alpha \operatorname{Re}\operatorname{Pr}}(\varphi_l''-\alpha^2\varphi_l)+\frac{ik_1}{\alpha \operatorname{Re}}\varphi_l'; \tag{1.19}$$

$$-c\varphi_s=-\frac{i}{\alpha \operatorname{Re}\operatorname{Pr}_s}(\varphi_s''-\alpha^2\varphi_s)+\frac{ik_1}{\alpha \operatorname{Re}}\varphi_s'. \tag{1.20}$$

Here and below the prime indicates differentiation with respect to y and $u = \partial\psi_0/\partial y$.

2. Solution of the System of Equations for the Amplitudes of the Perturbations. We assume that αRe is large. Then, Eq. (1.18) is a linear homogeneous ordinary differential equation with variable coefficients and the small parameter $(\alpha \operatorname{Re})^{-1}$ in front of the derivative of highest order. We seek the solution, following [2], in the form of a superposition of particular solutions

$$f(y)=C_1f_1(y)+C_2f_2(y)+C_3f_3(y)+C_4f_4(y),$$

where f_1 and f_2 are the solutions of the "nonviscous" equation, which is obtained from (1.18) by neglecting the viscosity

$$(u-c)(f''-\alpha^2 f)-u''f=0. \tag{2.1}$$

The solution of Eq. (2.1) can be found in the form of an expansion in powers of the difference $y-y_c$, where y_c is the point at which $u(y_c) = c$,

$$f_1=\sum_{k=1}^{\infty}a_k(y-y_c)^k,\quad f_2=2E_2f_1\ln(y-y_c)+\sum_{k=0}^{\infty}b_k(y-y_c)^k. \tag{2.2}$$

The coefficients in the expansions (2.2) satisfy the following recurrence relations:

$$a_1=1,\quad a_2=E_2,\quad a_{k+1}=E_{k+1}+\frac{1}{k(k+1)}\left[\alpha^2(E_{k-1}-E_0a_k)+\sum_{j=2}^{k}\gamma_j a_j\right],$$
$$k\geqslant 2,\quad b_0=1,\quad b_1=E_2,$$

$$b_{k+1}=\frac{k+2}{k}E_{k+2}+\frac{1}{k(k+1)}\left[\alpha^2(E_k-E_0b_k)+\sum_{j=1}^{k}\gamma_jb_j-2E_2\sum_{j=1}^{k+1}(2j-1)a_jE_{k-j+2}\right],\quad k\geqslant 1,$$
$$\gamma_j=\alpha^2E_{k-j}+(k+1)(k+2-2j)E_{k-j+2},\quad E_k=\frac{u^{(k)}(y_c)}{k!\,u'(y_c)}.$$

The other pair of independent particular solutions of Eq. (1.18) f_3 and f_4 are found in the form

$$f(y)=\exp\left(\int g_* dy\right),\quad g_*(y)=\sum_{m=0}^{m}(\alpha \operatorname{Re})^{\frac{1-m}{2}}g_m(y). \tag{2.3}$$

Substituting (2.3) into Eq. (1.18) leads to

$$g_0=\pm\sqrt{i(u-c)},\quad g_1=-\frac{5}{2}\frac{g_0'}{g_0}+\frac{k_1}{2},$$

as a result of which we obtain

$$f_{3,4}=(u-c)^{-5/4}\exp\left[\mp\int_{y_c}^{y}\sqrt{i\alpha \operatorname{Re}(u-c)}\,dy+\frac{k_1}{2}y\right].$$

The solutions f_3 and f_4 near $y = y_c$ are found directly from Eq. (1.18) by introducing the new variable $z = (y - y_c)/\varepsilon$, $\varepsilon = [\alpha \operatorname{Re} u'(y_c)]^{-1/3}$. If we seek the solution $f(y) = \chi(z)$

in the form of a series in powers of $\varepsilon, \chi(z)=\sum_{k=0}^{\infty} \varepsilon^k \chi^{(k)}$, then after equating the coefficients in front of the like powers of ε, we obtain

$$\chi_1^{(0)}=z, \quad \chi_2^{(0)}=1, \quad \chi_{3,4}^{(0)}=\int_{\pm\infty}^{z} dz \int_{\pm\infty}^{z} \sqrt{z} H_{1/3}^{(1,2)}\left[\frac{2}{3}(iz)^{3/2}\right] dz,$$

where $H^{(1)}{}_{1/3}$ and $H^{(2)}{}_{1/3}$ are Hankel functions. The asymptotic form of the Hankel functions permits identifying the solutions $f_{1,2,3,4}$ with the solutions $\chi_{1,2,3,4}$, as well as determining the required branch for circumscribing the point y_c

$$-7\pi/6 < \arg(y - y_c) < \pi/6.$$

Equation (1.19) is a linear inhomogeneous ordinary differential equation with the small parameters $(\alpha \text{ Re})^{-1}$ in front of the derivative of highest order. Its solution is sought in the form

$$\varphi_l(y) = C_5\varphi_{l1}(y) + C_6\varphi_{l2}(y) + \varphi_{l4}(y),$$

where $\varphi_{l1,2}$ is the fundamental system of solutions of the homogeneous equation, corresponding to (1.19),

$$(u-c)\varphi_l = -\frac{i}{\alpha \operatorname{Re} \operatorname{Pr}}(\varphi_l'' - \alpha^2\varphi_l) + \frac{ik_1}{\alpha \operatorname{Re}}\varphi_l'; \tag{2.4}$$

φ_{l4} is the particular solution of Eq. (1.19).

The solution of Eq. (2.4) is found in the form

$$\varphi_l(y) = \exp\left(\int h_* dy\right), \quad h_*(y) = \sum_{m=0}^{\infty} (\alpha \operatorname{Re})^{\frac{1-m}{2}} h_m(y). \tag{2.5}$$

Substitution of (2.5) into Eq. (2.4) leads to

$$h_0 = \pm\sqrt{i \operatorname{Pr}(u-c)}, \quad h_1 = -\frac{1}{2}\frac{h_0'}{h_0} + \frac{\operatorname{Pr} k_1}{2},$$

as a result of which we obtain

$$\varphi_{l1,2} = (u-c)^{-1/4} \exp\left[\mp\int_{y_c}^{y} \sqrt{i\alpha \operatorname{Re} \operatorname{Pr}(u-c)}\, dy + \frac{\operatorname{Pr} k_1}{2} y\right].$$

The particular solution of Eq. (1.19) can be represented by the asymptotic expansion

$$\varphi_{l4}(y) = \sum_{m=0}^{\infty} (\alpha \operatorname{Re})^{-m} \varphi_{l4}^{(m)}(y). \tag{2.6}$$

Substituting (2.6) into (1.19) and retaining the first term in the expansion, we obtain $\varphi_{l4} = t_0 f/(u - c)$.

The solutions of Eqs. (1.20) are sought using the same method as for the solutions of the homogeneous equation (2.4). As a result, the amplitude of the perturbation of the temperature of the solid metal is represented by the expression

$$\varphi_s(y) = C_7\varphi_{s1}(y) + C_8\varphi_{s2}(y), \quad \varphi_{s1,2} = \exp(\mp\sqrt{-i\alpha \operatorname{Re}\operatorname{Pr}_s c} + \operatorname{Pr}_s k_1/2)y.$$

3. Construction of Neutral Curves. We substitute (1.17) into the boundary conditions (1.5)-(1.16). After linearizing, we obtain the system of homogeneous algebraic equations. It follows from an analysis of this system that $p_1 = \rho_1 = t_1 = 0$. For the remaining unknowns we have the following conditions:

$$f'(0) + u'(0)n_0 = 0; \tag{3.1}$$

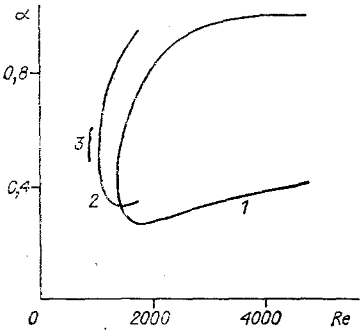

Fig. 2

$$f(0) = 0; \tag{3.2}$$

$$\varphi_l(0) + t'_{l0}(0)\, n_0 = 0; \tag{3.3}$$

$$\varphi_s(0) + t'_{s0}(0)\, n_0 = 0; \tag{3.4}$$

$$\varphi'_l(0) - \frac{\lambda_s}{\lambda_l}\varphi'_s(0) + \left[t''_{l0}(0) - \frac{\lambda_s}{\lambda_l} t''_{s0}(0)\right] n_0 = 0; \tag{3.5}$$

$$\varphi_s(-\infty) = 0; \tag{3.6}$$

$$f''(1) + \alpha^2 f(1) + [u''(1) - 2i\alpha u'(1)\,\mathrm{ctg}\,(2\beta)]n = 0; \tag{3.7}$$

$$f'(1) + u'(1)n = 0; \tag{3.8}$$

$$\varphi'_l(1) + [t''_{l0}(1) - i\alpha Q\cos\beta/\lambda]\, n = 0; \tag{3.9}$$

$$\varphi_l(1) + t'_{l0}(1)\, n = 0. \tag{3.10}$$

Equations (3.1)-(3.10) represent a system of homogeneous algebraic equations for the ten unknowns C_1-C_8, n_0, n. In order for the system to have a nontrivial solution, it is necessary and sufficient that its principal determinant vanish. After estimating the order of magnitude of the elements of the determinant, this condition assumes the form

$$\frac{f'_1(1) f_2(0) - f'_2(1) f_1(0)}{f'_1(1) f'_2(0) - f'_2(1) f'_1(0)} = \frac{f_3(0)}{f'_3(0)}. \tag{3.11}$$

Equation (3.11) is called the secular equation, and determines the characteristic values of the phase velocity c of the perturbations. The ratio $f_3(0)/f'_3(0)$ is expressed in terms of the tabulated Tit'ens functions [3], and the right side of Eq. (3.11) is determined with the help of expansions (2.2). If the imaginary part of c is set equal to zero, then relation (3.11) will be the equation for the neutral surface, separating the regions of stability and instability.

We are interested in the effect of the parameters q and δ of the problem on the stability. Selecting a specific metal and fixing the angle of incidence of the electrons β, we can obtain the neutral curve in the Re, α plane for different values of q. In Fig. 2 such curves, constructed from the numerical solution of Eq. (3.11), are shown for the flow of a film of melted iron with β = 5° (curve 1 corresponds to 10^9 W/m², curve 2 corresponds to $1.5\cdot 10^9$ W/m², and curve 3 corresponds to $2\cdot 10^9$ W/m²). The following values were used for the thermophysical characteristics [4, 5]: $\rho = 7230$ kg/m³, $c_l = 750$ J/(kg °K), $c_s = 700$ J/(kg·°K), $\lambda_l = 10$ W/(m·°K, $\lambda = 40$ W/(m·°K), $L_{melt} = 2.7\cdot 10^5$ J/kg, $L_{vap} = 6.3\cdot 10^6$ J/kg, $T_{melt} = 1810$°K, $T_b = 3300$°K, $\mu = 2.7\cdot 10^{-3}$ kg/(m·sec). The values of the remaining parameters used in the calculations were: $u_e = 25$ kV, $T_\infty = 300$°K.

For fixed q the magnitude of the Reynolds number is uniquely determined by the angle of inclination of the massive metal δ. Thus, the end points of the neutral curves (see Fig. 2), obtained for $q_1 = 10^9$ W/m², $q_2 = 1.5\cdot 10^9$ W/m², $q_3 = 2\cdot 10^9$ W/m², correspond to the angle of inclination δ = 90°, and the angles $\delta_1 = 14°$, $\delta_2 = 34°$, $\delta_3 = 79°$ correspond to the left-most point on the curves. This means that, for example, for $q = 10^9$ W/m^2 the flow will be stable relative to infinitesimal planar perturbations with an inclination of less than 14°. But, for $q > 2.01 \cdot 10^9$ W/m^2 (for $q = 2.01 \cdot 10^9$ W/m^2 the neutral curve degenerates into a point), the flow will be stable up to a vertical orientation of the film.

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CALCULATION OF FLOWS OF MELT IN AN AMPULE

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UDC 532.526.2

The problem of determining the dopant distribution in crystallization under conditions of reduced gravitational force is of current interest. The physical characteristics of such processes are examined in [1-3]. To solve this problem, one must know the flow velocity field of the melt. Schemes for solving this problem, primarily for moderate Reynolds and Marangoni numbers, are proposed in a number of papers [4, 5].

In this paper we propose an asymptotic scheme of stationary thermocapillary convection in a cylindrical ampule with large Reynolds and Marangoni numbers; this situation is realized in the presence of very high temperature differentials along the lateral wall of the ampule and low viscosity of the melt. The flow consists of a collection of Prandtl and Marangoni boundary layers, which join to the core of the flow. The axisymmetrical circulating flow in the core is calculated using the Prandtl–Batchelor scheme. The thermocapillary convection of the melt in the ampule is calculated using this scheme.

1. We are examining the problem of determining the thermocapillary convection velocity field of the melt in a cylindrical ampule with directed crystallization in the absence of gravity. The region of flow is illustrated in Fig. 1. The volume compression of semiconducting materials with melting gives rise to voids in the ampule, which are assumed to be distributed along the lateral wall of the ampule. The flow is assumed to be laminar, stationary, and axisymmetrical. The assumption of stationariness is explained by the fact that the time of crystallization of the entire ampule usually is several hours, so that the velocity of the crystallization front is of the order of 10^{-4} cm/sec, which is much lower than the velocity of thermocapillary convection with a very large temperature drop along the ampule.

Under the assumptions made above, the flow is described by the system of Navier–Stokes equations

$$uu_r + wu_z = -\frac{p_r}{\rho} + \nu\left(u_{rr} + \frac{1}{r}u_r - \frac{1}{r^2}u + u_{zz}\right), \quad (1.1)$$
$$uw_r + ww_z = -\frac{p_z}{\rho} + \nu\left(w_{rr} + \frac{1}{r}w_r + w_{zz}\right),$$
$$u_r + \frac{1}{r}u + w_z = 0,$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhanika i Tekhnicheskoi Fiziki, No. 2, pp. 105-110, March-April, 1984. Original article submitted February 4, 1983.

0021-8944/84/2502-0262$08.50